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DYNAMIC STABILITY OF VISCOELASTIC PLATES UNDER INCREASING COMPRESSING LOADS

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The dynamic stability problem of viscoelastic orthotropic and isotropic plates is considered in a geometrically nonlinear formulation using the generalized Timoshenko theory. The problem is solved by the Bubnov–Galerkin procedure combined with a numerical method based on quadrature formulas. The effect of viscoelastic and inhomogeneous properties of the material on the dynamic stability of a plate is discussed.

Key words: *Timoshenko theory, dynamic stability, viscoelasticity, Bubnov–Galerkin method.*

Introduction. The use of new composite materials in the design and development of strong, light-weight, and reliable structures calls for improved mechanical models of deformable solids and mathematical models for structure calculations taking into account the actual properties of structural materials. Numerous experimental studies have shown that most composite materials possess pronounced viscoelastic properties [2–6] and are inhomogeneous [6, 7].

The classical Kirchhoff–Love model is effective in solving some applied problems, but in most cases it fails to give adequate solutions [8]. This is true primarily for calculations of the dynamic stability of viscoelastic shells made of composite materials with heterogeneous anisotropic structure [3, 6, 9]. This formulation of elastic problems was considered in [7, 8, 10–13], where, however, only some properties of structural materials were taken into account.

In a number of papers, the viscoelastic properties of materials were taken into account only in the shear directions (see, e.g., [3]). In previous calculations, exponential kernels were used as relaxation kernels but they cannot provide an adequate description of the real processes occurring in shells and plates at the initial time [14]. The choice of exponential kernels in the calculations is not random. By differentiation, the resulting systems of integrodifferential equations were reduced to high-order ordinary differential equations, which, in most cases, were solved by the Runge–Kutta numerical method. Previously existing methods were not applicable for solving these problems with weakly singular kernels of the type of Koltunov, Rzhanitsyn, Abel, and Rabotnov kernels.

The numerical method developed in [1] using quadrature formulas has made it possible to solve systems of nonlinear integrodifferential equations with singular kernels. This method provides a high accuracy of calculations, is universal, and can be used to solve a wide class of dynamic problems of the theory of viscoelasticity. The results of [2, 9] obtained by this method agree well with experimental data.

It is worth noting that in contrast to the isotropic formulation of the dynamic problems of viscoelastic systems, where the integrodifferential equations contain only one relaxation kernel with three different rheological viscosity parameters, the orthotropic formulation includes seven different kernels and the number of different rheological parameters increases to 21, which leads to very intricate calculations.

The objective of the present paper is to study the dynamic stability of viscoelastic isotropic and orthotropic plates using various theories and to determine the applicability limits of these hypotheses in solving applied problems.

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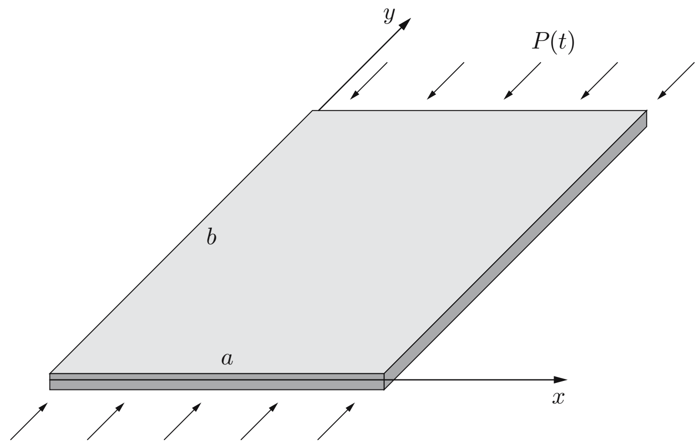

Fig. 1

Analysis of Deformation of a Viscoelastic Isotropic Rectangular Plate. We consider a viscoelastic isotropic rectangular plate with sides a and b subjected to unidirectional compression along the side a (Fig. 1). We assume that the compressing forces P increase linearly with time according to the law $P(t) = vt$ (v is the loading rate).

The relation among the stresses σ_x, σ_y, and τ_{xy} and the strains ε_x, ε_y, and γ_{xy} is given by [8, 9]

$$\sigma_x = \frac{E}{1-\mu^2}\,(1-R^*)(\varepsilon_x + \mu\varepsilon_y), \qquad x \leftrightarrow y, \qquad \tau_{xy} = \frac{E}{2(1+\mu)}\,(1-R^*)\gamma_{xy}, \tag{1}$$

where E is the modulus of elasticity, μ is Poisson's ratio, and R^* is an integral operator with the relaxation kernel $R(t)$: $R^*\varphi = \int\limits_0^t R(t-\tau)\varphi(\tau)\,d\tau$. Here and below, $x \leftrightarrow y$ denotes that the other relations are obtained by cyclic permutation of indices.

The geometrical relations among the strains ε_x^z, ε_y^z, and γ_{xy}^z and the rotations ψ_x and ψ_y are given by [7, 8]

$$\varepsilon_x^z = \varepsilon_x + z\,\frac{\partial \psi_x}{\partial x}, \qquad x \leftrightarrow y, \qquad \gamma_{xy}^z = \gamma_{xy} + z\Big(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\Big). \tag{2}$$

Here ε_x, ε_y, and γ_{xy} are determined from the relations [8]

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\Big[\Big(\frac{\partial w}{\partial x}\Big)^2 - \Big(\frac{\partial w_0}{\partial x}\Big)^2\Big], \qquad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\Big[\Big(\frac{\partial w}{\partial y}\Big)^2 - \Big(\frac{\partial w_0}{\partial y}\Big)^2\Big],$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\,\frac{\partial w_0}{\partial y}, \tag{3}$$

where $w_0 = w_0(x, y)$ is the initial deflection of the plate.

Substituting (1) and (2) into the equations of motion of the plate [7, 8] and introducing the stress functions $\Phi = \Phi(x, y, t)$, we obtain the system of integrodifferential equations that describes the dynamic stability of the viscoelastic isotropic plate [15] in terms of the deflection w, the stress function Φ, and the rotations ψ_x and ψ_y:

$$\frac{K^2E}{2(1+\mu)}\,(1-R^*)\Big(\nabla^2(w-w_0) + \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\Big) + L(w,\Phi) + \frac{q}{h} - P(t)\,\frac{\partial^2 w}{\partial x^2} - \rho\,\frac{\partial^2 w}{\partial t^2} = 0,$$

$$\frac{D}{h}\,(1-R^*)\Big(\frac{\partial^2 \psi_x}{\partial x^2} + \frac{1}{2}\,(1+\mu)\,\frac{\partial^2 \psi_y}{\partial x\,\partial y} + \frac{1}{2}\,(1-\mu)\,\frac{\partial^2 \psi_y}{\partial y^2}\Big)$$
$$- \frac{K^2E}{2(1+\mu)}\,(1-R^*)\Big(\frac{\partial\,(w-w_0)}{\partial x} + \psi_x\Big) - \rho\,\frac{h^2}{12}\,\frac{\partial^2 \psi_x}{\partial t^2} = 0, \qquad x \leftrightarrow y, \tag{4}$$

$$\frac{1}{E}\,\nabla^4\Phi = -\frac{1}{2}\,(1-R^*)[L(w,w)-L(w_0,w_0)].$$

Here h is the plate thickness, ρ is the density of the plate material, D is the flexural rigidity, and

$$L(w,w) = 2\Big[\frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - \Big(\frac{\partial^2 w}{\partial x\,\partial y}\Big)^2\Big], \quad L(w,\Phi) = \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - 2\,\frac{\partial^2 w}{\partial x\,\partial y}\,\frac{\partial^2 \Phi}{\partial x\,\partial y};$$

the coefficient K is determined in [7, 8].

Assuming that the edges of the plate are simply supported, we seek a solution of Eqs. (4) in the following form subject to the boundary conditions of the problem:

$$\begin{aligned}
w(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} w_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},\\
w_0(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} w_{0nm}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},\\
\psi_x(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{xnm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},\\
\psi_y(x,y,t) &= \sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{ynm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}.
\end{aligned} \tag{5}$$

Here $w_{nm} = w_{nm}(t)$, $\psi_{xnm} = \psi_{xnm}(t)$, and $\psi_{ynm} = \psi_{ynm}(t)$ are unknown functions of time.

Substituting the expressions for the deflections w and w_0 from (5) into the last equation of system (4) and equating the coefficients of the same trigonometrical functions on both sides of the resulting equation, we obtain the stress function

$$\begin{aligned}
\Phi(x,y,t) = E\sum_{i,r=1}^{N}\sum_{j,s=1}^{M}(1-R^*)(w_{ij}w_{rs}-w_{0ij}w_{0rs})\Big[C_{ijrs}\cos\frac{(i+r)\pi x}{a}\cos\frac{(j+s)\pi y}{b}\\
+ A_{ijrs}\cos\frac{(i+r)\pi x}{a}\cos\frac{(j-s)\pi y}{b} + D_{ijrs}\cos\frac{(i-r)\pi x}{a}\cos\frac{(j+s)\pi y}{b}\\
+ B_{ijrs}\cos\frac{(i-r)\pi x}{a}\cos\frac{(j-s)\pi y}{b}\Big] - \frac{P(t)y^2}{2},
\end{aligned} \tag{6}$$

where the dimensionless coefficients A_{ijrs}, B_{ijrs}, C_{ijrs}, and D_{ijrs} are determined as in [9].

Inserting (5) and (6) into the first three equations of system (4) and performing the Bubnov–Galerkin procedure, we obtain

$$\begin{aligned}
\frac{1}{S}\,\ddot{w}_{kl} - \Big(\frac{k}{\lambda}\Big)^2 t w_{kl} + \frac{5(1-\mu)}{4\pi^2}\,\delta^2\Big[\Big(\frac{k}{\lambda}\Big)^2 + l^2\Big](1-R^*)(w_{kl}-w_{okl})\\
+ \frac{5(1-\mu)}{4\pi^3}\,\delta^3(1-R^*)\Big[\Big(\frac{k}{\lambda}\Big)\psi_{xkl} + l\psi_{ykl}\Big]\\
= \frac{16\alpha_{kl}}{kl\pi^4}\,\frac{q}{P^*_{\rm cr}} - \frac{1}{P^*_{\rm cr}}\sum_{n,i,r=1}^{N}\sum_{m,j,s=1}^{M} a_{klnmijrs}w_{nm}(1-R^*)(w_{ij}w_{rs}-w_{oij}w_{ors}),\\
\frac{1}{S}\,\ddot{\psi}_{xkl} + \frac{3}{\pi^2}\,\delta^2\Big[\Big(\frac{k}{\lambda}\Big)^2 + \frac{1}{2}\,(1-\mu)l^2 + \frac{5(1-\mu)}{\pi^2}\,\delta^2\Big](1-R^*)\psi_{xkl}\\
+ \frac{3}{2\pi^2}\,\delta^2(1+\mu)\frac{k}{\lambda}\,l(1-R^*)\psi_{ykl} + \frac{15(1-\mu)k}{\pi^3\lambda}\,\delta^3(1-R^*)(w_{kl}-w_{okl}) = 0,\\
x \leftrightarrow y, \qquad k,l = 1,2,\ldots\ .
\end{aligned} \tag{7}$$

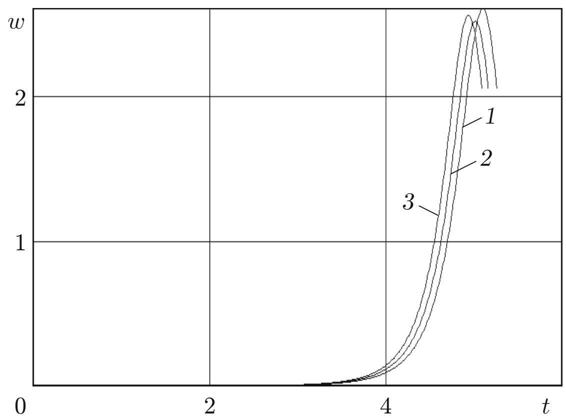

Fig. 2

In (7), we introduced the dimensionless quantities

$$\frac{w_{kl}}{h}, \quad \frac{w_{0kl}}{h}, \quad P^* = \frac{P}{E}\Big(\frac{b}{h}\Big)^2, \quad q^* = \frac{q}{E}\Big(\frac{b}{h}\Big)^4, \quad t^* = \frac{P}{P_{\rm cr}} = \frac{vt}{P_{\rm cr}} = \frac{\omega t}{\sqrt{S}} = \frac{P^*}{P^*_{\rm cr}},$$

$$\frac{\sqrt{S}}{\omega}\,R(t), \quad S = P^{*3}_{\rm cr}\Big(\frac{\pi c E h^3}{v b^4}\Big)^2, \quad P^*_{\rm cr} = \frac{P_{\rm cr}}{E}\Big(\frac{b}{h}\Big)^2 = \frac{\pi^2}{3(1-\mu^2)},$$

where $P_{\rm cr} = \pi^2 E h^2/(3(1-\mu^2)b^2)$ is the static critical load, $\omega = \sqrt{\pi^2 E h^2 P^*_{\rm cr}/(\rho b^4)}$ is the fundamental frequency of vibrations, $c = \sqrt{E/\rho}$ is the sound velocity in the plate material, S is the dimensionless parameter of the loading rate, $\lambda = a/b$, $\delta = b/h$, the coefficient $\alpha_{kl} = 1$ for odd k and l and if even one of these parameters is even, then $\alpha_{kl} = 0$, and the coefficient $a_{klnmijrs}$ is determined as in [9].

System (7) was integrated by the numerical method based on the quadrature formulas [1]. In the calculations, we used the simple but reasonably general weakly singular Koltunov–Rzhanitsyn kernel with three rheological parameters of the form $R(t) = A\,{\rm e}^{-\beta t}\,t^{\alpha-1}$ $(0 < \alpha < 1)$ [14] was used.

The calculation results are shown in Figs. 2–4. Here, as in [8], the criterion for determining the critical time and the critical load was the condition that the deflection should be smaller than the plate thickness is used. As the parameter determining the stability of the plate, we used the dynamic coefficient $K_{\rm dyn}$ equal to the ratio of the dynamic critical load to Euler's static load.

Figure 2 shows the calculation results for a square plate ($\lambda = 1$) with the initial deflection $w_0 = 10^{-4}$ in the absence of lateral load ($q = 0$). Curve 1 refers to calculations ignoring the viscoelastic properties of the plate material (elastic case) and curves 2 and 3 refer to the results obtained for the exponential relaxation kernel and the Koltunov–Rzhanitsyn kernel, respectively. An analysis shows that the values of $K_{\rm dyn}$ differ only slightly for these cases.

The effect of the parameter β on the behavior of the plate was also studied. The results obtained show that variations in the parameter β in the range $0 < \beta < 1$ have little effect on the critical time and load. This suggests that the equations with exponential relaxation kernels that are frequently used in solving dynamic problems of viscoelastic systems are inapplicable.

Figure 3 plots the function w of a viscoelastic square plate for various values of the geometrical parameter δ. For $\delta = 10$, 20, and 30 (curves 1, 2, and 3, respectively), the "critical" values of $K_{\rm dyn}$ are equal to 4.1, 4.4, and 4.5, respectively. We note that as the parameter δ increases, all curves are shifted toward the larger values of t and, hence, the dynamic coefficient increases. Here and below, the following data are used in the calculations: $A = 0.05$, $\beta = 0.25$, and $\alpha = 0.25$ (unless otherwise specified).

Figure 4 compares the results obtained for a square plate with $\delta = 20$ using various theories: the Kirchhoff–Love theory (curve 1), the Berger theory (curve 2), and the Timoshenko theory (curve 3). On can see that although the results obtained by the Kirchhoff–Love and Berger hypotheses differ qualitatively, the critical values of $K_{\rm dyn}$ are nevertheless equal and amount to 4.7. The critical value predicted by the Timoshenko theory is equal to 4.5.

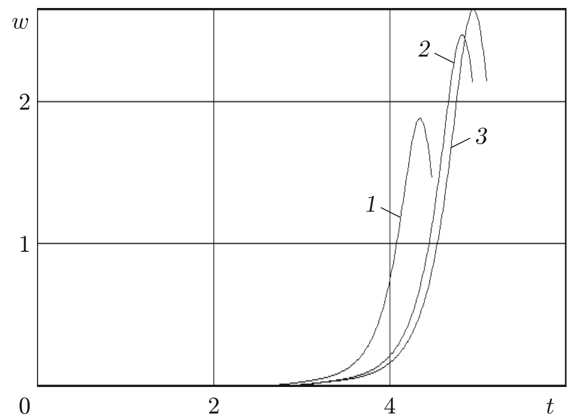

Fig. 3

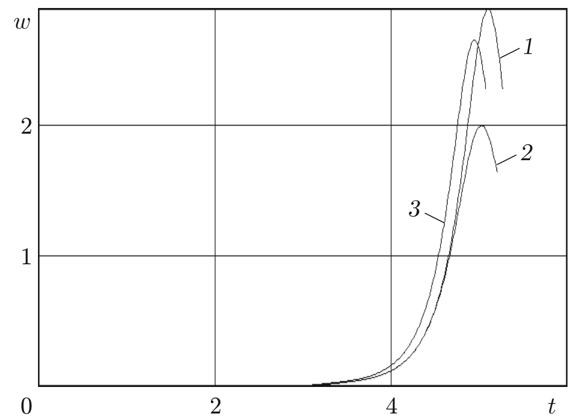

Fig. 4

TABLE 1

A	α	q	λ	w_0	S	δ	The values of K_{dyn} obtained in the solutions of the problem using the theories of		
							Berger	Kirchhoff–Love	Timoshenko
0.1	0.25	0	1	10^{-4}	1	20	4.87	4.45	4.35
0.03	0.25	0	1	10^{-4}	1	20	4.92	4.82	4.92
0.05	0.25	0	1	10^{-4}	1	20	4.90	4.75	4.77
0.08	0.25	0	1	10^{-4}	1	20	4.70	4.67	4.52
0.1	0.1	0	1	10^{-4}	1	20	3.32	3.05	3.15
0.1	0.5	0	1	10^{-4}	1	20	4.92	4.80	4.75
0.1	0.75	0	1	10^{-4}	1	20	4.92	4.85	4.85
0.1	0.25	1	1	10^{-4}	1	20	3.80	3.57	3.57
0.1	0.25	10	1	10^{-4}	1	20	2.10	1.97	1.97
0.1	0.25	0	2	10^{-4}	1	20	4.25	4.37	4.37
0.1	0.25	0	1	10^{-2}	1	20	3.45	3.17	3.22
0.1	0.25	0	1	10^{-1}	1	20	3.17	2.25	2.40
0.1	0.25	0	1	10^{-4}	0.1	20	8.15	7.47	7.25
0.1	0.25	0	1	10^{-4}	0.5	20	5.85	5.12	5.05
0.1	0.25	0	1	10^{-4}	10	20	3.05	2.67	2.65
0.1	0.25	0	1	10^{-4}	1	200	4.87	4.45	4.45
0.1	0.25	0	1	10^{-4}	1	10	4.87	4.45	4.10

Numerical solutions of the dynamic stability problem for a viscoelastic isotropic plate using various theories are listed in Table 1 for various geometrical and physical parameters of the plate. An analysis of these results shows that depending on the physicomechanical and geometrical parameters of the plate, the Timoshenko theory gives lower or higher values for the dynamic coefficient than the Berger and Kirchhoff–Love theories.

Deformation of Viscoelastic Orthotropic Rectangular Plates. We consider the same problem for a viscoelastic orthotropic plate. In this case, the relations among the stresses σ_x, σ_y, τ_{xy}, τ_{xz}, and τ_{yz} and the strains ε_x, ε_y, γ_{xy}, γ_{xz}, and γ_{yz} can be written in the following form [7, 14]:

$$\sigma_x = B_{11}(1 - R_{11}^*)\varepsilon_x + B_{12}(1 - R_{12}^*)\varepsilon_y, \qquad x \leftrightarrow y, \qquad 1 \leftrightarrow 2,$$
$$\tau_{xy} = 2B(1 - R^*)\gamma_{xy}, \qquad \tau_{xz} = 2B_{13}(1 - R_{13}^*)\gamma_{xz}, \qquad x \leftrightarrow y, \qquad 1 \leftrightarrow 2. \tag{8}$$

Here B_{ij} and B are elastic constants and R_{ij}^* and R^* are integral operators with the relaxation kernels $R_{ij}(t)$ and $R(t)$:

$$R^*\varphi = \int_0^t R(t-\tau)\varphi(\tau)\,d\tau, \qquad R_{ij}^*\varphi = \int_0^t R_{ij}(t-\tau)\varphi(\tau)\,d\tau, \qquad i = 1,2; \quad j = 1,2,3.$$

The relationship among the strains ε_x^z, ε_y^z, and γ_{xy}^z and the rotations ψ_x and ψ_y is given by (2). Substitution of (2) and (8) into the equations of motion of the plate yields the following system of integrodifferential equations for the displacements u and v, the deflection w, and the rotations ψ_x and ψ_y:

$$B_{11}(1-R_{11}^*)\,\frac{\partial\varepsilon_x}{\partial x}+B_{12}(1-R_{12}^*)\,\frac{\partial\varepsilon_y}{\partial x}+2B(1-R^*)\,\frac{\partial\gamma_{xy}}{\partial y}-\rho\,\frac{\partial^2 u}{\partial t^2}=0,$$

$$B_{22}(1-R_{22}^*)\,\frac{\partial\varepsilon_y}{\partial y}+B_{21}(1-R_{21}^*)\,\frac{\partial\varepsilon_x}{\partial y}+2B(1-R^*)\,\frac{\partial\gamma_{xy}}{\partial x}-\rho\,\frac{\partial^2 v}{\partial t^2}=0,$$

$$-2K^2\Big[B_{13}(1-R_{13}^*)\Big(\frac{\partial^2(w-w_0)}{\partial x^2}+\frac{\partial\psi_x}{\partial x}\Big)+B_{23}(1-R_{23}^*)\Big(\frac{\partial^2(w-w_0)}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\Big)\Big]$$

$$-\frac{\partial}{\partial x}\Big\{\frac{\partial w}{\partial x}\,[B_{11}(1-R_{11}^*)\varepsilon_x+B_{12}(1-R_{12}^*)\varepsilon_y]+2B\,\frac{\partial w}{\partial y}\,(1-R^*)\gamma_{xy}\Big\} \tag{9}$$

$$-\frac{\partial}{\partial y}\Big\{\frac{\partial w}{\partial y}\,[B_{22}(1-R_{22}^*)\varepsilon_y+B_{21}(1-R_{21}^*)\varepsilon_x]+2B\,\frac{\partial w}{\partial x}\,(1-R^*)\gamma_{xy}\Big\}-\frac{q}{h}+P(t)\,\frac{\partial^2 w}{\partial x^2}+\rho\,\frac{\partial^2 w}{\partial t^2}=0,$$

$$\frac{B_{11}h^2}{12}\,(1-R_{11}^*)\,\frac{\partial^2\psi_x}{\partial x^2}+\frac{B_{12}h^2}{12}\,(1-R_{12}^*)\,\frac{\partial^2\psi_y}{\partial x\,\partial y}+\frac{Bh^2}{6}(1-R^*)\Big(\frac{\partial^2\psi_x}{\partial y^2}+\frac{\partial^2\psi_y}{\partial x\,\partial y}\Big)$$

$$-\,2K^2B_{13}(1-R_{13}^*)\Big(\frac{\partial\,(w-w_0)}{\partial x}+\psi_x\Big)-\frac{\rho h^2}{12}\,\frac{\partial^2\psi_x}{\partial t^2}=0,\qquad x\leftrightarrow y,\quad 1\leftrightarrow 2.$$

Here ε_x, ε_y, and γ_{xy} are determined from relations (3).

Substituting the expressions for the deflections and rotations (5) and the relations

$$u(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$

$$v(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}v_{nm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}$$

for the displacements u and v, respectively, into system (9) and applying the Bubnov–Galerkin procedure, we obtain the following system of ordinary nonlinear integrodifferential equations

$$\frac{1}{S}\,\ddot{u}_{kl}+\Big[\frac{6\Delta\delta^2k^2}{\pi^2\lambda^2\eta}\,(1-R_{11}^*)+\frac{6g\delta^2l^2(1-\mu_1\mu_2)}{\pi^2\eta}\,(1-R^*)\Big]u_{kl}=-\frac{6\delta^2kl}{\pi^2\lambda\eta}\,[\Delta\mu_2(1-R_{12}^*)+g(1-\mu_1\mu_2)(1-R^*)]v_{kl}$$

$$+\,\frac{6\Delta\delta}{\pi^3\lambda^3\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}ni^2\mu_{nik}\beta_{mjl}(1-R_{11}^*)(w_{nm}w_{ij}-w_{0nm}w_{0ij})$$

$$+\,\frac{6\Delta\delta\mu_2}{\pi^3\lambda\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}mij\gamma_{nik}\alpha_{mjl}(1-R_{12}^*)(w_{nm}w_{ij}-w_{0nm}w_{0ij})$$

$$+\,\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}(mij\gamma_{nik}\alpha_{mjl}+nj^2\mu_{nik}\beta_{mjl})(1-R^*)(w_{nm}w_{ij}-w_{0nm}w_{0ij}),$$

$$\frac{1}{S}\,\ddot{v}_{kl}+\Big[\frac{6\delta^2l^2}{\pi^2\Delta\eta}\,(1-R^*_{22})+\frac{6g\delta^2k^2(1-\mu_1\mu_2)}{\pi^2\lambda^2\eta}\,(1-R^*)\Big]v_{kl}$$

$$=-\frac{6\delta^2kl}{\pi^2\Delta\lambda\eta}\,[\mu_1(1-R^*_{21})+\Delta g(1-\mu_1\mu_2)(1-R^*)]u_{kl}$$

$$+\frac{6\delta}{\Delta\pi^3\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}mj^2\beta_{nik}\mu_{mjl}(1-R^*_{22})(w_{nm}w_{ij}-w_{0nm}w_{0ij})$$

$$+\frac{6\delta\mu_1}{\Delta\pi^3\lambda^2\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}nij\alpha_{nik}\gamma_{mjl}(1-R^*_{21})(w_{nm}w_{ij}-w_{0nm}w_{0ij})$$

$$+\frac{6g\delta(1-\mu_1\mu_2)}{\pi^3\lambda^2\eta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}(nij\alpha_{nik}\gamma_{mjl}+n^2j\beta_{nik}\gamma_{mjl})(1-R^*)(w_{nm}w_{ij}-w_{0nm}w_{0ij}),$$

$$\frac{1}{S}\,\ddot{w}_{kl}-\Big(\frac{k}{\lambda}\Big)^2tw_{kl}+\frac{3K^2\delta^2(1-\mu_1\mu_2)}{2\pi^4\eta}\,[4g_{13}\pi^2k^2(1-R^*_{13})w_{kl}+4g_{13}\pi\delta k(1-R^*_{13})\psi_{xkl}$$

$$+g_{23}\lambda^2l^2(1-R^*_{23})w_{kl}+2g_{23}\lambda\delta l(1-R^*_{23})\psi_{ykl}]+\frac{6\lambda^2\delta^2}{\Delta\pi^4\eta}\,(1-R^*_{22})(w_{kl}-w_{0kl})$$

$$=\sum_{n,i,k=1}^{N}\sum_{m,j,l=1}^{M}w_{nm}\Big[\frac{6\Delta\delta}{\pi^3\eta}\,a_{nmijkl}(1-R^*_{11})+\frac{3g\lambda^2\delta(1-\mu_1\mu_2)}{2\pi^5\eta}\,c_{nmijkl}(1-R^*)+\frac{3\mu_1\lambda^2\delta}{2\Delta\pi^5\eta}\,f_{nmijkl}(1-R^*_{21})\Big]u_{ij}$$

$$+\sum_{n,i,k=1}^{N}\sum_{m,j,l=1}^{M}w_{nm}\Big[\frac{3\Delta\mu_2\lambda\delta}{\pi^4\eta}\,b_{nmijkl}(1-R^*_{12})+\frac{3g\lambda\delta(1-\mu_1\mu_2)}{\pi^4\eta}\,d_{nmijkl}(1-R^*)+\frac{3\lambda^3\delta}{4\Delta\pi^6\eta}\,e_{nmijkl}(1-R^*_{22})\Big]v_{ij}$$

$$-\sum_{n,i,k,r=1}^{N}\sum_{m,j,l,s=1}^{M}w_{nm}\Big[\frac{3\Delta}{16\eta}\,h_{nmijklrs}(1-R^*_{11})-\frac{3\Delta\mu_2\lambda^2}{64\pi^2\eta}\,p_{nmijklrs}(1-R^*_{12})\qquad(10)$$

$$+\frac{3g\lambda^2(1-\mu_1\mu_2)}{32\pi^2\eta}\,g_{nmijklrs}(1-R^*)+\frac{3\lambda^4}{256\Delta\pi^4\eta}\,q_{nmijklrs}(1-R^*_{22})-\frac{3\mu_1\lambda^2}{64\Delta\pi^2\eta}\,r_{nmijklrs}(1-R^*_{21})\Big](w_{ij}w_{rs}-w_{0ij}w_{0rs})$$

$$-\sum_{n,i,k=1}^{N}\sum_{m,j,l=1}^{M}\Big[\frac{3\lambda^3\delta}{4\Delta\pi^6\eta}\,mj\beta_{nik}\alpha_{mjl}(1-R^*_{22})+\frac{3\mu_1\lambda\delta}{\Delta\pi^4\eta}\,ni\alpha_{nik}\beta_{mjl}(1-R^*_{21})\Big](w_{nm}w_{ij}-w_{0nm}w_{0ij})$$

$$+\sum_{n,i,k=1}^{N}\sum_{m,j,l=1}^{M}w_{nm}\Big[\frac{6\Delta\mu_2\lambda\delta}{\pi^4\eta}\,s_{nmijkl}(1-R^*_{12})+\frac{3\lambda^3\delta}{2\Delta\pi^6\eta}\,x_{nmijkl}(1-R^*_{22})\Big](w_{ij}-w_{0ij})$$

$$-\frac{3\lambda^2\delta^2l}{\Delta\pi^4\eta}\,(1-R^*_{22})v_{kl}-\frac{6\mu_1\lambda\delta^2k}{\Delta\pi^3\eta}\,(1-R^*_{21})u_{kl}+\frac{96\alpha_{kl}(1-\mu_1\mu_2)}{\pi^6\eta kl}\,q,$$

$$\frac{1}{S}\,\ddot{\psi}_{xkl}+\frac{6\Delta\delta^2k^2}{\pi^2\eta}\,(1-R^*_{11})\psi_{xkl}+\frac{3\Delta\mu_2\lambda\delta^2kl}{\pi^3\eta}\,(1-R^*_{12})\psi_{ykl}$$

$$+\frac{3g\lambda^2\delta^2l^2(1-\mu_1\mu_2)}{2\pi^4\eta}\,(1-R^*)\psi_{xkl}+\frac{3g\lambda\delta^2kl(1-\mu_1\mu_2)}{\pi^3\eta}\,(1-R^*)\psi_{ykl}$$

$$+\frac{72K^2g_{13}\delta^3k(1-\mu_1\mu_2)}{\pi^3\eta}\,(1-R^*_{13})w_{kl}+\frac{72K^2g_{13}\delta^4(1-\mu_1\mu_2)}{\pi^4\eta}\,(1-R^*_{13})\psi_{xkl}=0,$$

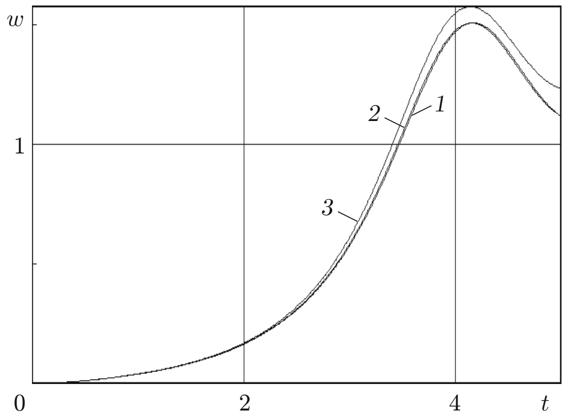

Fig. 5

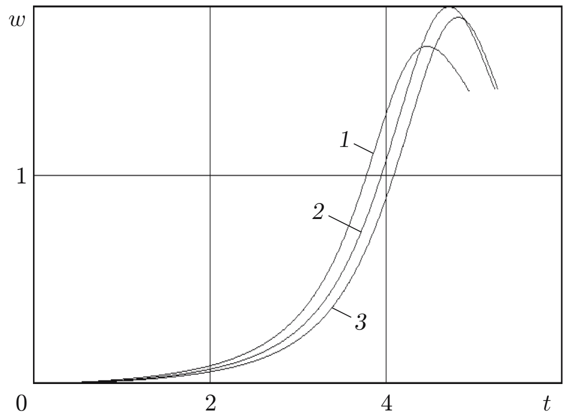

Fig. 6

$$\frac{1}{S}\,\ddot{\psi}_{ykl} + \frac{3\lambda^2\delta^2 l^2}{2\Delta\pi^4\eta}\,(1-R^*_{22})\psi_{ykl} + \frac{3\mu_1\lambda\delta^2 kl}{\Delta\pi^3\eta}\,(1-R^*_{21})\psi_{xkl}$$

$$+\,\frac{3g\lambda\delta^2 kl(1-\mu_1\mu_2)}{\pi^3\eta}\,(1-R^*)\psi_{xkl} + \frac{6g\delta^2 k^2(1-\mu_1\mu_2)}{\pi^2\eta}\,(1-R^*)\psi_{ykl}$$

$$+\,\frac{36K^2 g_{23}\lambda\delta^3 l(1-\mu_1\mu_2)}{\pi^4\eta}\,(1-R^*_{23})w_{kl} + \frac{72K^2 g_{23}\delta^4(1-\mu_1\mu_2)}{\pi^4\eta}\,(1-R^*_{23})\psi_{ykl} = 0,$$

$$k, l = 1, 2, \ldots\ .$$

In (10), we introduced the dimensionless parameters

$$\frac{w_{kl}}{h},\quad \frac{w_{0kl}}{h},\quad \frac{u_{kl}}{h},\quad \frac{v_{kl}}{h},\quad P^* = \frac{P}{\sqrt{E_1E_2}}\Big(\frac{b}{h}\Big)^2,\quad q^* = \frac{q}{\sqrt{E_1E_2}}\Big(\frac{b}{h}\Big)^4,$$

$$t^* = \frac{P}{P_{\rm cr}} = \frac{vt}{P_{\rm cr}} = \frac{\omega t}{\sqrt{S}} = \frac{P^*}{P^*_{\rm cr}},\quad \frac{\sqrt{S}}{\omega}\,R(t),\quad \frac{\sqrt{S}}{\omega}\,R_{ij}(t),\quad i = 1,2,\quad j = 1,2,3,$$

$$S = P^{*3}_{\rm cr}\Big(\frac{\pi c\sqrt{E_1E_2}\,h^3}{vb^4}\Big)^2,\quad P^*_{\rm cr} = \frac{P_{\rm cr}}{\sqrt{E_1E_2}}\Big(\frac{b}{h}\Big)^2 = \frac{\pi^2}{6(1-\mu_1\mu_2)}\,\eta,$$

where $P_{\rm cr} = \pi^2\sqrt{E_1E_2}h^2\eta/(6(1-\mu_1\mu_2)b^2)$ is the static critical load, $c = \sqrt{\sqrt{E_1E_2}/\rho}$, $\omega = \sqrt{\pi^2\sqrt{E_1E_2}\,h^2P^*_{\rm cr}/(\rho b^4)}$, $\Delta = \sqrt{E_1/E_2}$, $g = G/\sqrt{E_1E_2}$, $\eta = 1 + \Delta\mu_2 + 2(1-\mu_1\mu_2)g$, E_1 and E_2 are the moduli of elasticity, μ_1 and μ_2 are Poisson's ratios satisfying the condition $E_1\mu_2 = E_2\mu_1$, and the remaining coefficients of this system are given in [9]. System (10) was also integrated by the numerical method based on quadrature formulas [1].

Figures 5 and 6 give the calculation results for the deformation of a viscoelastic orthotropic square plate taking into account the shear strain and rotary inertia for $\Delta = 1.5$ and $\delta = 20$.

Figure 5 shows curves $w(t)$ for various viscoelastic properties of the material of the structure. Curve 1 refers to the case where viscoelastic properties are ignored ($A = A_{ij} = 0$, where $i = 1, 2$ and $j = 1, 2, 3$ is the elastic case), curve 2 refers to the case (which is used in [3] to study viscoelastic orthotropic structures $A = A_{13} = A_{23} = 0$) where the viscoelastic properties are taken into account only for the shear directions, and curve 3 refers to the case where the viscoelastic properties are taken into account for all directions simultaneously ($A = A_{ij} = 0.1$, where $i = 1, 2$ and $j = 1, 2, 3$). One can see from Fig. 5 that the results obtained for the elastic case are very close to those obtained for the case where the viscoelastic properties are taken into account only for the shear directions. Moreover, accounting for the viscoelastic properties of the material leads to an earlier intense increase in the deflection and, hence, to smaller critical values of $K_{\rm dyn}$.

Figure 6 shows curves of $w(t)$ for various degrees of anisotropy of the plate material. One can see that an increase in the parameter Δ, which determines the degree of anisotropy, (curve 1 refers to $\Delta = 1$, curve 2 to $\Delta = 2$, and curve 3 to $\Delta = 3$) leads to a later intense increase in the deflection and, hence, to a higher critical value of $K_{\rm dyn}$. Similar results were observed in experiments performed on composite structures [8]. This supports the validity of the chosen method and the reliability of the results obtained.

Conclusions. The present studies of nonlinear dynamic stability problems for isotropic and orthotropic viscoelastic plates showed that:

— In the analysis of viscoelastic structures, the viscoelastic properties of the material should be taken into account not only for the shear direction but also for the other directions simultaneously;

— As the relaxation kernels that take into account the viscoelastic properties of structural materials, one should use Koltunov–Rzhanitsyn type kernels, which contain a sufficient number of rheological parameters;

— Depending on the geometrical and physical parameters of plates, one should choose appropriate theories that agree with experimental results: the Berger theory, the classical Kirchhoff–Love theory, or the refined Timoshenko theory.

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